



## Assignment 3

Welcome to the third assignment of the lecture *3D Vision and Deep Learning*. **Please read all instructions carefully!** The goal of this assignment is to get familiar with the projective images of points and lines as well as with some further aspects of rotations. Submission is due on Monday, May 6th, 2024 at 2pm. Please note that late assignments will receive zero (0) marks, so you are strongly encouraged to start the assignment early.

**Exercise 1.** (2 points) Show that the vanishing line  $\mathbf{l}_\infty = (0, 0, 1)^T$  is mapped by a homography  $\mathbf{H}$  onto itself if and only if  $\mathbf{H}$  is an *affine mapping*.

**Exercise 2.** (1 point) Show that every *point transformation* of the form

$$\mathbf{H} = \mathbf{H}_A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{pmatrix},$$

where  $\mathbf{H}_A$  is an arbitrary affine mapping, maps the straight line  $\mathbf{l} = (l_1, l_2, l_3)^T$  onto the vanishing line  $\mathbf{l}_\infty$ .

**Exercise 3.** (3 points) Given are the points  $\mathbf{p} = (1, 1, 0)$ ,  $\mathbf{q} = (-1, -1, -1) \in \mathbb{R}^3$  and the plane  $\pi$  defined by the three points  $(1, 0, 0)$ ,  $(0, 1, 0)$ ,  $(0, 0, 1) \in \mathbb{R}^3$ .

1. (1 point) Determine a homogeneous representation of the plane  $\pi$ .
2. (1 point) Determine the Plücker coordinates of the straight line  $L$  connecting the points  $\mathbf{p}$  and  $\mathbf{q}$ .
3. (1 point) Determine the intersection of the straight line  $L$  with the plane  $\pi$ .

**Exercise 4.** (4 points) Given are the quaternions

$$\mathbf{q}_1 = \left[ \frac{\sqrt{2}}{2}, \begin{pmatrix} 0 \\ \frac{\sqrt{2}}{2} \\ 0 \end{pmatrix} \right], \mathbf{q}_2 = \left[ 2, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right], \mathbf{q}_3 = \left[ \frac{\sqrt{2}}{2}, \begin{pmatrix} 0 \\ 0 \\ \frac{\sqrt{2}}{2} \end{pmatrix} \right], \mathbf{q}_4 = \left[ \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \right]$$

Give the rotation matrices for all *unit quaternions* (1 point) and calculate (3 points)  $\mathbf{q}(t) = \mathbf{q}_1(\mathbf{q}_1^{-1}\mathbf{q}_3)^t$ , for  $t = 1/4$ .

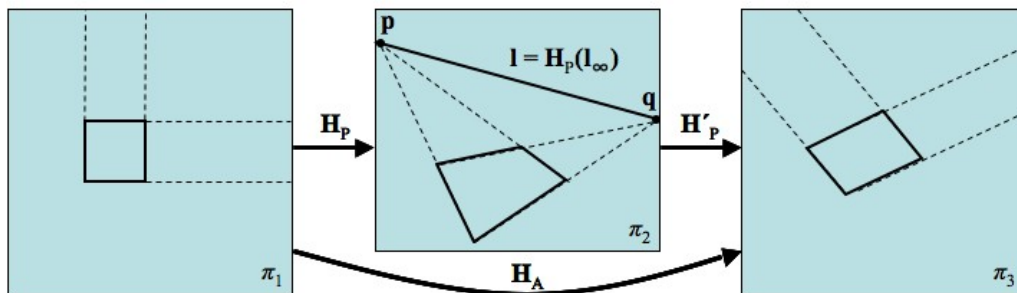


**Exercise 5.** (2 points) Write a python programm, that reads a  $3 \times 3$  matrix from file and checks wether it is a rotation matrix or not. Your program should output the corresponding *quaternion* if the matrix is a rotation matrix. Test your program with the following two matrices which are also defined in `mat1.dat` and `mat2.dat`.

$$\mathbf{M}_1 = \begin{pmatrix} 1 & 1 - \sqrt{3} & 1 + \sqrt{3} \\ 1 + \sqrt{3} & 1 & 1 - \sqrt{3} \\ 1 - \sqrt{3} & 1 + \sqrt{3} & 1 \end{pmatrix}$$

$$\mathbf{M}_2 = \begin{pmatrix} \frac{1}{3}(1 + \sqrt{2}) & \frac{1}{3} - \frac{\sqrt{2}}{6}(1 + \sqrt{3}) & \frac{1}{3} - \frac{\sqrt{2}}{6}(1 - \sqrt{3}) \\ \frac{1}{3} - \frac{\sqrt{2}}{6}(1 - \sqrt{3}) & \frac{1}{3}(1 + \sqrt{2}) & \frac{1}{3} - \frac{\sqrt{2}}{6}(1 + \sqrt{3}) \\ \frac{1}{3} - \frac{\sqrt{2}}{6}(1 + \sqrt{3}) & \frac{1}{3} - \frac{\sqrt{2}}{6}(1 - \sqrt{3}) & \frac{1}{3}(1 + \sqrt{2}) \end{pmatrix}.$$

**Exercise 6.** (8 points) A projective mapping  $\mathbf{H}_P$  maps the vanishing line  $\mathbf{l}_\infty = (0, 0, 1)^T$  in the Euclidean plane  $\pi_1$  onto the finite line  $\mathbf{l}$  in the plane  $\pi_2$ . If a projective mapping  $\mathbf{H}'_P$  remaps  $\mathbf{l}$  onto  $\mathbf{l}_\infty$  in the plane  $\pi_3$ , than the mapping  $\mathbf{H}_A = \mathbf{H}'_P \circ \mathbf{H}_P$  between  $\pi_1$  and  $\pi_3$  (see figure below) is an affine mapping as it keeps  $\mathbf{l}_\infty$  fix.



Based on the image  $\mathbf{l} = (l_1, l_2, l_3)^T$  of the vanishing line  $\mathbf{l}_\infty$ , the affine properties (parallelism, ratio of areas) of a scene can be reconstructed. For this one determines a projective mapping  $\mathbf{H}'_P$ , that remaps  $\mathbf{l}$  back onto  $\mathbf{l}_\infty$ . For  $l_3 \neq 0$  with any affine mapping  $\mathbf{H}_A$  it is clear that

$$\mathbf{H}'_P = \mathbf{H}_A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ l_1 & l_2 & l_3 \end{pmatrix} \implies \mathbf{H}'_P{}^{-T} \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \mathbf{l}_\infty.$$

The image  $\mathbf{l} = (l_1, l_2, l_3)^T$  of the vanishing line  $\mathbf{l}_\infty$  results from the intersections  $\mathbf{p}, \mathbf{q}$  of the images of two families of parallel lines (see top figure).

Write an OpenCV application for affine image rectification, that

- (4 points) allow to select two pairs of straight lines that are parallel in the image. Use the two intersections  $\mathbf{p}, \mathbf{q}$  to determine the image  $\mathbf{l}$  of the vanishing line  $\mathbf{l}_\infty$ . Use e.g. the function `HoughLines` to detect straight lines in images and implement a suitable metaphor to interactively select two parallel lines.



2. (4 points) uses the intersections  $\mathbf{p}$ ,  $\mathbf{q}$  of the pairs of straight lines determined in 1. to calculate the image  $\mathbf{l}$  of the vanishing line  $\mathbf{l}_\infty$  and determines a projective mapping  $\mathbf{H}'_p$ . This mapping should be used to transform the original image. Finally, the transformed (rectified) image should be shown.

The two steps of the described affine image rectification process, namely 1. *determination of two pairs of parallel lines* and 2. *determination and application of  $\mathbf{H}'_p$* , are again shown in the three images (original image, image with detected lines, rectified image) below. Note the parallelism of the lines in the rectified image. For example images see the archive `FloorsAndBuildings.zip`.

