



Assignment 1

Welcome to the first assignment of the lecture *3D Vision and Deep Learning* in summer semester 2024. **Please read all instructions carefully!** The goal of this assignment is a brief repetition of the most important basic math concepts needed to understand some of the theoretical background of *3D Vision and Deep Learning*.

Submission is due on Monday, April 22, 2024 at 2pm. Please note that late assignments will receive zero (0) marks, so you are strongly encouraged to start the assignment early.

Exercise 1 (4 points). Consider the three vectors

$$\mathbf{x} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} 5 \\ 6 \end{bmatrix}, \quad \mathbf{z} = \begin{bmatrix} 7 \\ 8 \end{bmatrix}.$$

1. Determine the *inner product* $\langle \mathbf{x}, \mathbf{y} \rangle = \mathbf{x}^\top \mathbf{y}$ of \mathbf{x} and \mathbf{y} .
2. Determine the *outer product* $\mathbf{x} \otimes \mathbf{y} = \mathbf{x} \mathbf{y}^\top$ of \mathbf{x} and \mathbf{y} .
3. Determine $(\mathbf{x} \otimes \mathbf{y}) \mathbf{z}$
4. What is the *rank* of $\mathbf{x} \otimes \mathbf{y}$?

Exercise 2 (4 points). Consider the 2×2 matrices

$$R_1 = \begin{bmatrix} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{bmatrix}, \quad R_2 = \begin{bmatrix} -\sin \alpha & \cos \alpha \\ \cos \alpha & \sin \alpha \end{bmatrix}, \quad R_3 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}.$$

1. Determine the determinants of the matrices.
2. Are the matrices orthonormal?
3. Are the matrices invertible? If so, give the inverse. If not, explain why.
4. What is the difference between R_1 and R_2 ?

Exercise 3 (4 points). Show that for any nonzero vector $\mathbf{u} = (u_1, u_2, u_3)^\top \in \mathbb{R}^3$, the rank of the *skew-symmetric* matrix (sometimes also written as $\hat{\mathbf{u}}$ or \mathbf{u}_\times)

$$[\mathbf{u}]_\times = \begin{bmatrix} 0 & -u_3 & u_2 \\ u_3 & 0 & -u_1 \\ -u_2 & u_1 & 0 \end{bmatrix}$$

is always two. That is, the three row (or column) vectors span a two-dimensional subspace of \mathbb{R}^3 . *Hint:* If we fix \mathbf{u} the cross product $\mathbf{u} \times \mathbf{v}$ can be represented by linear map from \mathbb{R}^3 to \mathbb{R}^3 represented by the matrix $[\mathbf{u}]_\times$.



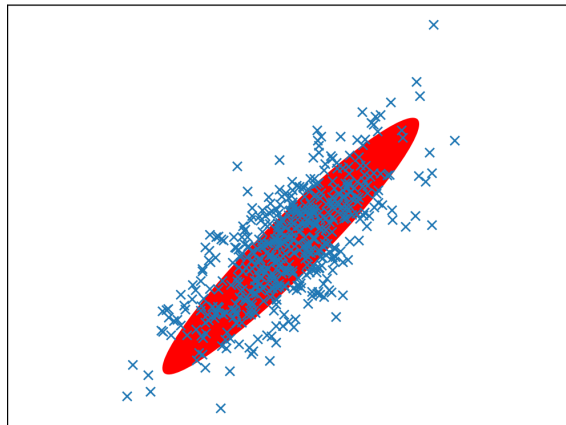
Exercise 4 (4 points). Show that for the skew symmetric matrix $[\mathbf{u}]_{\times}$ defined in exercise 3 it holds

$$[\mathbf{u}]_{\times}^2 = \mathbf{u} \otimes \mathbf{u} - \langle \mathbf{u}, \mathbf{u} \rangle \cdot I \quad \text{and} \quad [\mathbf{u}]_{\times}^3 = -\langle \mathbf{u}, \mathbf{u} \rangle \cdot [\mathbf{u}]_{\times} \quad (\text{i.e. } \|\mathbf{u}\| = 1 \implies [\mathbf{u}]_{\times}^3 = -[\mathbf{u}]_{\times})$$

Exercise 5 (6 points). What are the derivatives of

1. $y = \frac{1}{1+e^{-x}}$ with respect to x
2. $y = |x|$ with respect to x
3. $y = \mathbf{w}^{\top} \mathbf{x}$ ($\mathbf{x}, \mathbf{w} \in \mathbb{R}^n$) with respect to \mathbf{x}
4. $y = \mathbf{w}^{\top} \mathbf{x}$ ($\mathbf{x}, \mathbf{w} \in \mathbb{R}^n$) with respect to \mathbf{w}
5. $\mathbf{y} = M\mathbf{x}$ ($M \in \mathbb{R}^{m \times n}, \mathbf{x} \in \mathbb{R}^n$) with respect to \mathbf{x}
6. y_i with respect to m_{ij} , where $\mathbf{y} = M\mathbf{x}$ ($M \in \mathbb{R}^{m \times n}, \mathbf{x} \in \mathbb{R}^n$)

Exercise 6 (4 points). Write a simple python programm `covmat.py` that uses `numpy` to compute the covariance matrix of the pointset that is given by the file `covtest.dat`. Your programm should also visualize the ellipse centered at the mean of the data which have the principal axes given by the eigenvectors and eigenvalues of the covariance matrix (see figure below).



Exercise 7 (4 points). Write a simple python program `scale.py` that uses OpenCV to load, display, manipulate, and save an image. Your program should be able to

1. (1 point) display the loaded image,
2. (2 points) scale the image by a factor $s \in [0.1, 2]$. The scale factor s should be changeable by a slider (provided by OpenCV) in increments of 0.1. Use another Slider to change the different possible interpolation methods.
3. (1 point) and save the scaled image.